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Analysis of Neutronic Space-Time Effects in Large LMFBR's with Thermal-Hydraulic Feedback Using FX2-TH Nuclear Reactor Kinetics Code and Mode Expansion of the Neutron Flux Shape Function

by

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ANALYSIS OF NEUTRONIC SPACE-TIME EFFECTS IN LARGE LMFBR's WITH THERMAL-HYDRAULIC FEEDBACK USING FX2-TH NUCLEAR REACTOR KINETICS CODE AND MODE EXPANSION OF THE NEUTRON FLUX SHAPE FUNCTION

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ABSTRACT

For a delayed critical transient in an LMFBR, the dependence of the local flux tilt on the degree of heterogeneity in the core configuration was examined. Using a mode expansion of the neutron flux shape function, decoupling the resulting mode kinetics equations and utilizing the prompt jump approximation, an expression for the i th harmonic local flux tilt was developed. According to this expression, the i th harmonic local flux tilt $\tau_i(\vec{r},t)$ is inversely proportional to the sum of the eigenvalue separation between the i th harmonic and the fundamental mode and the delayed neutron fraction $(\frac{1}{k_i} - \frac{1}{k_o} + \beta)$ and is directly proportional to a time dependent factor $T_i(t)$ and a space dependent factor $G_i(\vec{r})$. The modified eigenvalue separation $(\frac{1}{k_i} - \frac{1}{k_o} + \beta)$ and $G_i(\vec{r})$ are known when the eigenvalues and eigenfunctions of the unperturbed reactor are available. Thus if $T_i(t)$, which depends upon the perturbation, varies little in comparison with the modified eigenvalue separation from one core configuration to another, a simple measure of the i th harmonic local flux tilt is feasible. This flatness criterion for $T_i(t)$ has been investigated by inserting 2¢/sec ramp reactivity in three core configurations. During the transient, the shape function was calculated with the FX2-TH code and $\tau_i(\vec{r},t)$ and $T_i(t)$ were extracted using mode expansion of the shape function. The results of this analysis are presented

INTRODUCTION

The flux tilt or the change in the neutron flux shape function may be significant during some delayed critical transients in Large Heterogeneous Liquid-Metal Fast Breeder Reactors[1] (LMFBR's). The shape function may be calculated accurately using the two-dimensional FX2-TH code[2] which takes into account thermal-hydraulic feedback effects. However, during the design stage when several reactor configurations with differing degrees of heterogeneity and differing fuel compositions may be under consideration, transient analysis studies with the FX2-TH code could be too expensive. Hence, a simple measure of the flux tilt (albeit approximate) would be useful in the transient analysis of heterogeneous fast breeder reactors. It has been shown previously[3,4,5] that, in the asymptotic steady state, such a measure can be provided by the fundamental to the first harmonic eigenvalue separation if the asymptotic neutron flux may be approximated by a linear combination of the fundamental and the first harmonic eigenfunctions of the unperturbed reactor. During the transient, however, the expansion of the neutron flux shape function in terms of the fundamental and the first harmonic eigenfunctions alone may not be adequate. Hence, it is assumed that the shape function may be approximated by an expansion in terms of the fundamental mode and few lower harmonics.

The flux tilt $\tau(\vec{r},t)$ is defined such that i 'th mode contribution to it is equal to the product of the time dependent coefficient $b_i(t)$ in the expansion of the shape function and a space dependent term $G_i(\vec{r})$ which depends upon the fundamental and i th harmonic eigenfunctions. Following the methods of Wade and Rydin[3] and Kaplan[4] and using suitable approximations, it is shown in the Appendix that $b_i(t)$ is inversely proportional to a quantity which is the sum of the i th harmonic eigenvalue separation from the fundamental and the effective delayed neutron fraction (β) with a time dependent proportionality factor $T_i(t)$. This modified eigenvalue separation reduces to the asymptotic form when the i th harmonic eigenvalue separation is much larger than β. The modified eigenvalue separations and $G_i(\vec{r})$ are essentially known once the modes of the unperturbed steady state reactor are evaluated. If the variation of $T_i(t)$ in going from one reactor configuration to another is relatively smooth in comparison with the corresponding variation in the eigenvalue separation or $G_i(\vec{r})$, a simple measure of the local flux tilt may be feasible. Since evaluation of $T_i(t)$ in the presence of space dependent thermal-hydraulic feedback requires knowledge of the shape function itself, the following empirical procedure was adopted. After evaluating the modes of the initial steady state reactor, at full power, the transient was followed using the FX2-TH code. The modal flux tilt $\tau_i(\vec{r},t)$ was evaluated during the transient and $T_i(t)$ was extracted from it by using a suitable normalization for the mode functions used in the evaluation of $G_i(\vec{r})$.

THEORY

The mode representation of a neutron flux shape function may be written as

$$\phi(\vec{r},E,t) = \sum_i b_i(t)\, \psi_i(\vec{r},E) \tag{1}$$

where $\psi_i(\vec{r},E)$ are the eigenfunctions of the unperturbed reactor in the initial steady state and $b_i(t)$ are the coefficients of the expansion.

Let the local power density in a small volume ΔV around the point $\vec{r}$ be

$$P(\vec{r},t) = \frac{1}{\Delta V} \int_{\Delta V} M(\vec{r} + \vec{r}^{\,'},E,t)\, \phi(\vec{r} + \vec{r}^{\,'},E,t)\, d\vec{r}^{\,'}\, dE \tag{2}$$

and local relative power density be

$$R(\vec{r},t) = \frac{P(\vec{r},t)}{P(\vec{r},o)} \tag{3}$$

where $M(t) = M_o + \delta M_o(t)$ is the fission operator.

Correspondingly, the total power and the relative total power are given by

$$P(t) = \int_{\text{Reactor}} M(\vec{r},E,t)\, \phi(\vec{r},E,t)\, d\vec{r}\, dE \tag{4}$$

and

$$R(t) = \frac{P(t)}{P(o)} \tag{5}$$

Then, the local flux tilt in a small volume ΔV at point $\vec{r}$ may be defined as

$$\tau(r,t) = \frac{R(\vec{r},t)}{R(t)} - 1. \tag{6}$$

On substitution of the expressions for $\phi(\vec{r},E,t)$, $R(\vec{r},t)$ and $R(t)$, Eq. 6 gives

$$\tau(\vec{r},t) = \sum_i \frac{b_i(t)}{b_o(t)} G_i(\vec{r},t) - [1 - G_o(\vec{r},t)]$$

$$= \sum_i \tau_i(\vec{r},t) - [1 - G_o(\vec{r},t)] \tag{7}$$

where

$$G_i(\vec{r},t) = \frac{\int_{\Delta V} M(\vec{r} + \vec{r}',E,t)\ \psi_i(\vec{r} + \vec{r}',E)\ dE\ d\vec{r}'}{\int_{\Delta V} M_o(\vec{r} + \vec{r}',E)\ \psi_o(\vec{r} + \vec{r}',E)\ dE\ d\vec{r}'} \tag{8}$$

An equation for the expansion coefficient $b_i(t)$ is derived in the Appendix and is given by

$$b_i(t) = \frac{T_i(t)}{(\frac{1}{k_i} - \frac{1}{k_o} + \beta)} \tag{9}$$

where k_o and k_i are the multiplication constants in the fundamental mode and the i th mode respectively, $T_i(t)$ is a time dependent factor depending on the mode dependent transition reactivity and precursor concentrations (see Eqs. A-14, A-17 and A-30 in the Appendix), and β is the effective delayed neutron fraction. For the delayed critical transients under consideration in this study, where the net reactivity inserted is not large, the coefficient of the fundamental mode $b_o(t)$ in the expansion of the shape function may be assumed to be close to unity and since $M(\vec{r},E,t)$ and $M_o(\vec{r},E,0)$ may not differ significantly from each other the time dependence in $G_i(\vec{r},t)$ may be ignored. Hence the i th harmonic local flux tilt may be written as

$$\tau_i(\vec{r},t) \approx \frac{T_i(t)}{(\frac{1}{k_i} - \frac{1}{k_o} + \beta)} G_i(\vec{r}) \tag{10}$$

METHODOLOGY

Two-dimensional time-dependent multigroup diffusion equations are solved in FX2-TH code[2] by use of the improved quasistatic method.[6] The thermal-hydraulic model option used in this study solves the heat conduction equations in a one-dimensional fuel pellet, gap, clad and coolant configuration. The code takes into account feedback effects from changes in both the average fuel and the average coolant temperatures. The neutronics solutions are based on an eight energy group cross section set for four temperatures and on six delayed neutron family data derived from the ENDF/B-IV data files.

Due to limitations of the FX2-TH code, it is assumed that boiling does not occur at any point in the transient, that there is no distortion of geometry and that fuel does not expand. The transient modeled for this study is due to withdrawal, at power, of a single control rod located in the farthest ring from the center of the reactor. The calculations were done in triangular geometry with half core symmetry and the half-core was divided into about 60 thermal-hydraulic regions.

For modal analysis of the neutron flux shape function during the transient, eigenfunctions of the initial steady state reactor are needed. A macroscopic cross section file for the materials in each thermal-hydraulic region was generated using the converged steady state fuel and coolant temperature distributions obtained in the steady state option of the FX2-TH code. With the half core symmetry in triangular geometry, these cross sections were used in the RINGIT code[7] to calculate the fundamental mode and the harmonics of the initial steady state reactor. The RINGIT code makes use of a random guess for the initial real and adjoint neutron flux distributions and calls the DIF3D code[8] to calculate both the real and adjoint eigenfunctions for each harmonic. When a harmonic has converged to the desired accuracy, that harmonic is stripped from the flux guess and the code will proceed to the calculation of the next harmonic using the stripped flux guess.

CORE CONFIGURATIONS

The three core configurations used in this study are shown in Fig. 1. The size of the reactors was 1000 MW(electric) and the degree of heterogeneity ranged from homogeneous, through tightly coupled heterogeneous to loosely coupled heterogeneous. In the tightly coupled core, one row of internal blankets separates the core regions and in the loosely coupled core, two rows of internal blankets separate the core regions. Both the heterogeneous cores have a blanket at the core center. All cores were enrichment zoned, two zones in the case of the homogeneous configuration and three zones in the case of the heterogeneous configurations. The general reactor data and the fuel and blanket pin descriptions on which the assembly descriptions are based are given in Table I.

DISCUSSION OF RESULTS

For each core configuration, the initial steady state neutron flux, fuel and coolant temperatures were calculated with the steady state option in the FX2-TH code. Four lower harmonics besides the fundamental mode were calculated at the steady state for the configurations and they were numbered sequentially in the direction of increasing eigenvalue separation. A schematic diagram of the nodal lines and the eigenvalue separation from the fundamental mode of each harmonic is given in Table II. In the three configurations considered here,

TABLE I

Design Characteristics

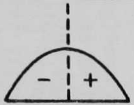
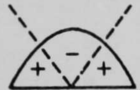
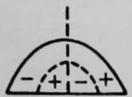
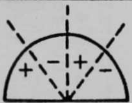
General Reactor Data	
Reactor power, MW(electric)	1000
Reactor power, MW(thermal)	2740
Thermal efficiency, %	36.5
Core height, cm	101.6 (40 in.)
Axial blanket thickness, cm	38.1 (15 in.)
Reactor outlet temperature, °C	499 (930°F)
Reactor ΔT, °C	156 (280°F)

Fuel Pin Design Data		**Blanket Assembly and Pin Design**	
Fuel material	Pu-UO_2	Fuel material	UO_2
Fuel/clad bond	Helium	Fuel/clad bond	Helium
Pin o.d., mm	7.4 (0.290 in.)	Duct outside flat-to-flat, cm	15.34 (6.040 in.)
Cladding thickness, mm	0.35 (0.014 in.)	Duct wall thickness, mm	3.58 (0.141 in.)
p/d ratio	1.186	Number of pins	127
Wire spacer diameter, mm	1.35 (0.053 in.)	Pin o.d., mm	11.94 (0.470 in.)
Plenum length, cm	122 (48 in.)	Cladding thickness, mm	0.38 (0.015 in.)
Fuel smeared density, % TD	90	p/d ratio	1.070
Linear power, W/cm		Fuel smeared density, % TD	90
Nominal peak	459 (14.0 kW/ft)		
Average	295 (9.0 kW/ft)		
Fuel Assembly Design		**Blanket Volume Fractions**	
Number of pins	271	Fuel (smeared)	0.5592
Assembly pitch, cm	16.03 (6.310 in.)	Structure	0.1656
Duct outside flat-to-flat, cm	15.34 (6.040 in.)	Sodium	0.2752
Duct wall thickness, mm	3.58 (0.141 in.)		
Interassembly gap, mm	6.86 (0.270 in.)		
Nozzle-to-nozzle Δp, kPa	620 (90 psi)		
Peak coolant velocity, m/s	7.9 (26.0 ft/s)		
Core Volume Fractions			
Fuel (smeared)	0.4238		
Structure	0.1969		
Sodium	0.3793		

the first and second harmonics have angular nodes and the third harmonic has a radial node. The fourth harmonic in the loosely coupled core has one angular and one radial node while the fourth harmonic in both tightly coupled and homogeneous cores has angular nodes only. For each harmonic, the eigenvalue separation decreases as the degree of heterogeneity increases. For example, the eigenvalue separation of the first harmonic in the loosely coupled core is smaller approximately by factors of 1.5 and 3.0 than the first harmonic eigenvalue separation in the tightly coupled core and the homogeneous core respectively. The contribution of each harmonic to the local flux tilt was calculated at chosen intervals during the transient.

An overpower transient was initiated in each core configuration by the withdrawal of a single control rod at a rate adjusted so as to insert 2¢/sec ramp reactivity. Due to thermal-hydraulic feedback, the net reactivity insertion rate was lower in each core configuration as shown in Fig. 2. This decrease can be accounted for by the large negative reactivity contribution from the Doppler feedback and a relatively smaller positive contribution from the coolant density feedback. The net reactivity inserted after 15 sec. was 6.0¢, 6.9¢ and 7.8¢ respectively in the homogeneous core, the tightly coupled core and the loosely coupled core. The relative reactor power is shown in Fig. 3 as a function of time for each core configuration. The time dependent local

TABLE II. Eigenvalue Separations in the Three Configurations

Harmonic i	Schematic Diagram of Nodal Lines in Half Core	Loosely Coupled Core		Tightly Coupled Core		Homogeneous Core	
		$\left(\frac{1}{k_i} - \frac{1}{k_o}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_o} + \beta\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_o}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_o} + \beta\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_o}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_o} + \beta\right)$
1		0.0098	0.0133	0.01465	0.0182	0.0309	0.0345
2		0.0254	0.029	0.0431	0.0467	0.0720	0.0756
3		0.0435	0.0471	0.09163	0.0952	0.1004	0.104
4		0.0739	0.0775				
4				0.1018	0.1053	0.133	0.1363

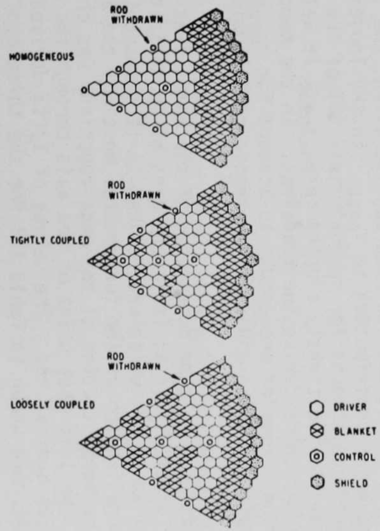


FIG. 1 1000 MWE CORE CONFIGURATIONS

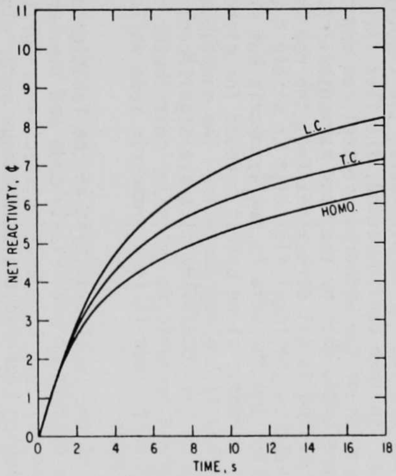


FIG. 2 TIME RESPONSE OF THE NET REACTIVITY AS PREDICTED BY SPACE-TIME KINETICS FOR A 2¢/ SEC. RAMP REACTIVITY INSERTION

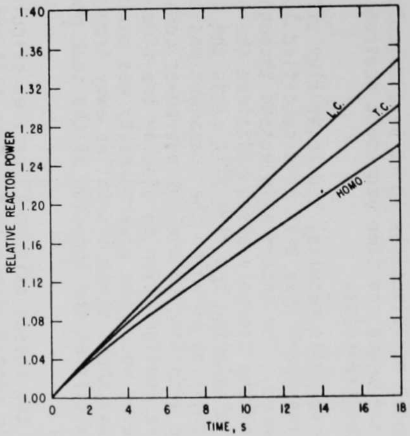


FIG. 3 TIME RESPONSE OF THE RELATIVE REACTOR POWER AS PREDICTED BY SPACE-TIME KINETICS FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

flux tilt and the modal contributions to it were calculated with the available mode functions and the space and time dependent neutron flux shape function calculated during the transient.

For displaying mode analysis results, the local flux tilt along an axis at a specified time and the local flux tilt at a specified location as a function of time were considered. The chosen axis passed through the core center and through the center of the assembly which contained the withdrawn control rod. The time chosen for displaying the flux tilt along the axis was 15 sec. since it corresponded to the time at which the transient was terminated in the loosely coupled core according to the 30% overpower constraint. The chosen location in each core configuration to display the time dependence of the local flux tilt was the point where the power density was maximum and it was found that this location was about three assemblies away from the withdrawn control rod. The local flux tilt at the location of the peak power density was designated as τ_{peak}.

The flux tilt along the chosen axis at 15 sec. as calculated with the FX2-TH code and the harmonic contributions to the flux tilt are shown in Figs. 4, 5 and 6 respectively for the loosely coupled core, the tightly coupled core and the homogeneous core. In each configuration, the flux tilt along the axis is a maximum at the location of the withdrawn control rod and at this point the flux tilt is not represented well by the four harmonics. About three assemblies away and beyond in all directions from the rod location, the lower four harmonics represented the local flux tilt to within $\pm$ 20% of the FX2-TH value. The flux tilt along the axis is not symmetric but skewed towards the right half and first harmonic alone cannot account for either the magnitude or the shape of the flux tilt along the axis. The contributions of the third and fourth harmonics to the local flux tilt are significant only around the core centre. The second harmonic is the major contributor to the neutron flux pile-up in the right half since it is a symmetric mode while the first harmonic is an antisymmetric mode.

The time dependence of the local flux tilt at the location of peak power density $\tau_{peak}(t)$, as calculated with FX2-TH code and the harmonic contributions to it are shown in Figs. 7, 8 and 9 respectively for the loosely coupled core, the tightly coupled core and the homogeneous core. In each core configuration, after about 2 sec. into the transient τ_{peak} and harmonic contributions to it vary linearly with time. In each configuration, the first and second harmonics are the major contributors to τ_{peak}. In the loosely coupled core, the lower four harmonics account for approximately 90% of the τ_{peak} throughout the transient. In the tightly coupled core, τ_{peak} is well represented by the four harmonics during the transient. In the homogeneous core, the first and second harmonics are enough to account for τ_{peak} during the transient and inclusion of third and fourth harmonic contributions over-predicts the τ_{peak}.

For the transient considered, the time dependence of the total flux tilt $\tau(\vec{r},t)$ and the i th harmonic flux tilt $\tau_i(\vec{r},t)$ is independent of normalization of the mode functions but $T_i(t)$ and $G_i(\vec{r})$ which are related to it through Eq. (10) are dependent on the normalization of the fundamental mode and the i th harmonic eigenfunction. The normalization of the mode functions was chosen such that $G_i(\vec{r})$ was unity at the left most point on the axis through the center of the core and the withdrawn control rod. The values of $T_i(t)$ derived using this normalization at t=15 sec. are shown in Table III for the three configurations.

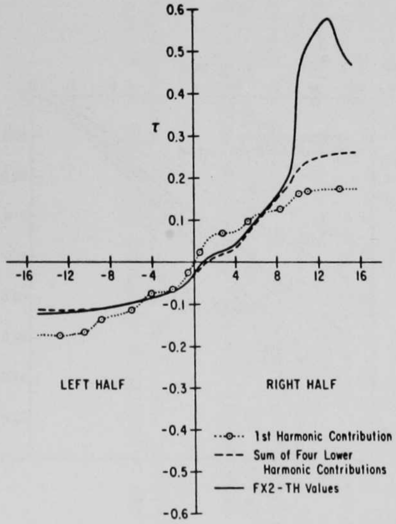


FIG. 4 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE LOOSELY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

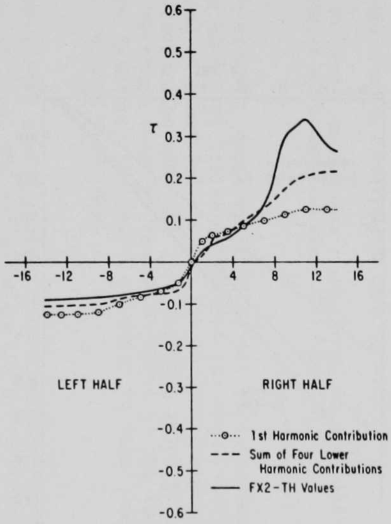


FIG. 5 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE TIGHTLY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

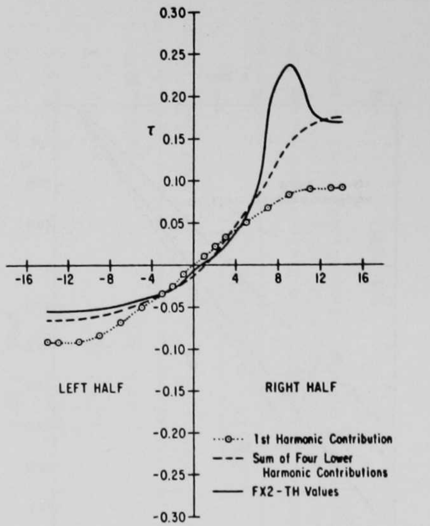


FIG. 6 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE HOMOGENEOUS CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

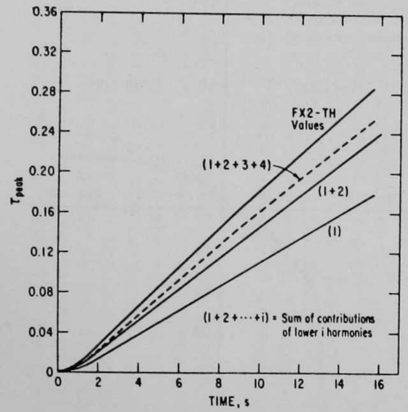


FIG. 7 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE LOOSELY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

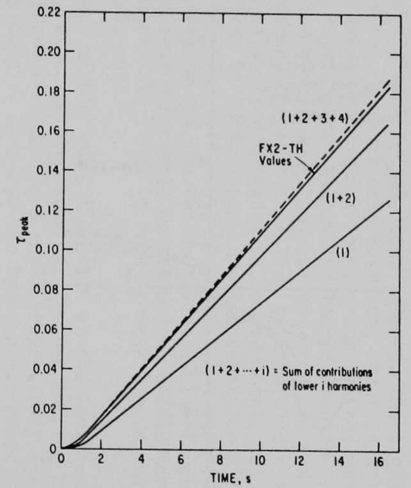


FIG. 8 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE TIGHTLY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

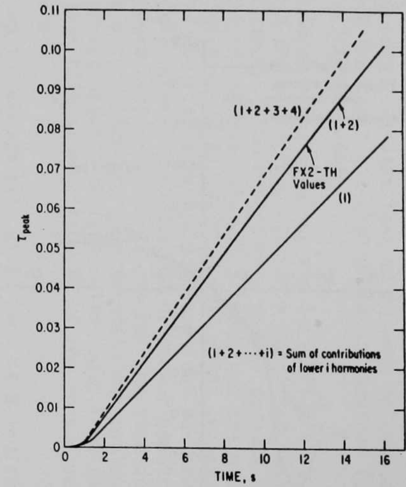


FIG. 9 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE HOMOGENEOUS CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

Table III

$T_i(t)$ Values in three configurations at t=15 sec for 2¢/sec Ramp Reactivity Insertion ($G_i(\vec{r})$ are normalized such that they are unity at the left hand edge of the axis which passes through the withdrawn control rod)

Harmonic i	Loosely Coupled Core	$T_i(t)^a$ at t = 15 sec Tightly Coupled Core	Homogeneous Core
1	2.35×10^{-3}	2.28×10^{-3}	3.11×10^{-3}
2	1.7×10^{-3}	2.175×10^{-3}	1.6×10^{-3}
3	6.5×10^{-4}	7.18×10^{-4}	10.7×10^{-4}
4	0.8×10^{-3} [b]	3.48×10^{-3}	3.88×10^{-3}

[a] These values are based on normalization used for $G_i(\vec{r})$.

[b] 4th harmonic in loosely coupled core is of a different type than the 4th harmonic in tightly coupled and homogeneous cores (see Table II).

Even though i th harmonic quantity $T_i(t)$ is not constant between configurations, it can be seen that its variation between them is not large. A variation in $T_i(t)$ of about 65% occured for the third harmonic in the homogeneous core in comparison with the corresponding value in the loosely coupled core. It is to be noted that the fourth harmonic in the loosely coupled core is of a different type from the fourth harmonic in other configurations and hence cannot be compared with them.

CONCLUSION

For the three LMFBR core configurations considered and for the delayed critical transient studied in them, the variation of the perturbation dependent quantity $T_i(t)$ for the i th harmonic at any time t from one core configuration to another was not found to be large. Hence, for the same transient initiated in a new core configuration, whose heterogeneity falls in the range considered here, the local flux tilt may be estimated harmonic by harmonic when the eigenfunctions and eigenvalues of the new configuration in the unperturbed state are available. The flatness criterion for $T_i(t)$ between configurations cannot be guaranteed, in general, for all possible transients and must be ascertained before the relationship between the i th harmonic local flux tilt and its modified eigenvalue separation from the fundamental is applied.

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APPENDIX

SPACE-TIME KINETICS EQUATIONS FOR THE λ-MODE EXPANSION COEFFICIENTS

The time-dependent multigroup kinetics equations[9] are

$$V \frac{\partial\phi(\vec{r},t)}{\partial t} + L\phi(\vec{r},t) - (1-\beta)\, \chi_p\, F^+\phi(\vec{r},t) - \sum_{i=1}^{6} \lambda_i\, (\chi_{di} C_i(\vec{r},t))$$

$$= Q\,(\vec{r},t) \qquad \text{(A-1)}$$

and

$$\frac{\partial}{\partial t}\,(\chi_{di} C_i(\vec{r},t) + \lambda_i \chi_{di} C_i(\vec{r},t) = \beta_i\, \chi_{di}\, F^+\phi(\vec{r},t) \qquad i = 1,6 \qquad \text{(A-2)}$$

Here ϕ is a column vector of neutron group flux, $C_i(\vec{r},t)$ is the concentration of the precursors of type i at $\vec{r}$ and time t, V is a diagonal matrix with elements $\frac{1}{v_g}$ where v_g is the neutron speed in group g, χ_p is a column vector of the prompt neutron spectrum, χ_{di} is a column vector of the i th type delayed neutron spectrum, F^+ is a row vector with elements $\nu\Sigma_{fg}$ where

ν is the number of neutrons emitted in fission and Σ_{fg} is the macroscopic fission cross section in group g, L is the loss operator, λ_i is the decay constant of the i th type precursors and β_i is the corresponding delayed neutron fraction.

Consider the unperturbed reactor in steady state with $k_{eff} = k_o = 1$ and with the unperturbed flux $\phi_o(\vec{r},o)$ being given by a constant multiple of the fundamental mode $\psi_o(\vec{r})$ of the λ-mode eigenvalue problem.

$$L_o \psi_n = \frac{1}{k_n} M_o \psi_n \tag{A-3}$$

where L_o is the time independent loss operator, M_o is the corresponding production operator and k_n is the eigenvalue of the n th λ-mode.

Let perturbations $\delta L_o(\vec{r},t)$ and $\delta M_o(\vec{r},t)$ be introduced in the reactor such that

$$L(\vec{r},t) = L_o(\vec{r}) + \delta L_o(\vec{r},t) \tag{A-4}$$

and

$$M(\vec{r},t) = M_o(\vec{r}) + \delta M_o(\vec{r},t). \tag{A-5}$$

Defining

$$M = \chi_p F^+(\vec{r},t) \tag{A-6}$$

and

$$\chi_{di} = \chi^i \chi_p \tag{A-7}$$

where χ^i is a diagonal matrix with elements $\chi^i_{g,g} = \chi_{di,g}/\chi_{p,g}$, the kinetics equations may be written in the form.

$$V \frac{\partial \phi(\vec{r},t)}{\partial t} + L\phi(\vec{r},t) - [(1-\beta) + \sum_{i=1}^{6} \beta_i \chi^i] M\phi(\vec{r},t)$$

$$+ \sum_{i=1}^{6} \beta_i \chi^i M \phi(\vec{r},t) - \sum_{i=1}^{6} \lambda_i \chi_{di} C_i(\vec{r},t) = Q(\vec{r},t) \tag{A-8}$$

and

$$\frac{\partial}{\partial t} (\chi_{di} C_i(\vec{r},t)) + \lambda_i \chi_{di} C_i(\vec{r},t) = \beta_i M\phi(\vec{r},t) + \beta_i (\chi^i - I) M\phi(\vec{r},t) \tag{A-9}$$

for i=1,6

where I is the unit matrix.

It can be seen that in the steady state at t=o and for Q=o, the kinetics equations (A-8 and A-9) reduce to the λ-mode eigenvalue problem with k_o=1 if the difference between the prompt and delayed neutron Spectra may be neglected (i.e., χ^i=I).

Expanding the perturbed flux $\phi(\vec{r},t)$ in λ-modes of Eq. (A-3) we have

$$\phi(\vec{r},t) = \sum_j A_j(t)\ \psi_j(\vec{r}) \qquad \text{(A-10)}$$

Selective substitution of Eq. (A-10) for $\phi(\vec{r},t)$ in Eqs. (A-8 and A-9), multiplying the resulting equations on the left by the adjoint function ψ_n^*, integrating over the whole volume and noting k_o=1 gives [3,4]

$$\sum_j \Lambda_{nj} \frac{\partial A_j(t)}{\partial t} + \left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right) A_n(t) - \sum_{i=1}^{6} \lambda_i C_{ni}(t)$$

$$= \rho_n(t) - \sum_{i=1}^{6} \beta_i\ S_{ni}(t) + Q_n(t) \qquad \text{(A-11)}$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i\ C_{ni}(t) - \beta_i A_n(t) = \beta_i \delta_{ni}(t) + \beta_i S_{ni}(t) \qquad \text{for } i=1,6 \qquad \text{(A-12)}$$

where use is made of the inner product notation $\int x^T y d\vec{r} = (x,y)$ with x^T being the transpose of x and

$$\Lambda_{nj} = \frac{(\psi_n^*,\ V\ \psi_j)}{(\psi_n^*,\ M_o \psi_n)}\ , \qquad \text{(A-13)}$$

$$\rho_n(t) = \frac{\left(\psi_n^*,\ [-\ \delta L_o + \delta M_o - \sum_{i=1}^{6} \beta_i\ (\chi^i - I)\ \delta M_o]\ \phi(\vec{r},t)\right)}{(\psi_n^*,\ M_o \psi_n)}\ , \qquad \text{(A-14)}$$

$$S_{ni}(t) = \frac{(\psi_n^*,\ \chi^i \delta M_o \phi)}{(\psi_n^*,\ M_o \psi_n)}\ , \qquad \text{(A-15)}$$

$$\delta_{ni}(t) = \frac{(\psi_n^*, (\chi^i - I)\, M_o\phi)}{(\psi_n^*, M_o\psi_n)}, \tag{A-16}$$

$$C_{ni}(t) = \frac{(\psi_n^*, \chi_{di}C_i(\vec{r},t))}{(\psi_n^*, M_o\psi_n)}, \tag{A-17}$$

and

$$Q_n(t) = \frac{(\psi_n^*, Q(\vec{r},t))}{(\psi_n^*, M_o\psi_n)} \tag{A-18}$$

For $t \to \infty$, if an asymptotic steady state is reached such that $\rho_n(\infty)$ and $S_{ni}(\infty)$ are independent of time and if the difference between the prompt and delayed neutrons may be neglected, we get the asymptotic value for $A_n(\infty)$ which is

$$A_n(\infty) = \frac{\rho_n(\infty)}{(\frac{1}{k_n} - \frac{1}{k_o})} \tag{A-19}$$

DECOUPLED QUASISTATIC SYNTHESIS APPROXIMATION

In the quasistatic[6] approximation, we may represent $\phi(\vec{r},t)$ exactly as

$$\phi(\vec{r},t) = N(t)\Psi(\vec{r},t) \tag{A-20}$$

where $N(t)$ is the amplitude function and $\Psi(\vec{r},t)$ is the shape function which is constrained by

$$\int_{\text{Reactor}} \psi^*(\vec{r},o)\ V\ \Psi(\vec{r},t) = 1.0 \text{ for all } t. \tag{A-21}$$

In the quasistatic synthesis approximation, the shape function is written as

$$\Psi(\vec{r},t) = \sum_j b_j(t)\ \psi_j(\vec{r}) \tag{A-22}$$

All coefficients $b_j(t)$ are not independent since $\Psi(\vec{r},t)$ obeys Eq. (A-21). Hence, knowledge of $N(t)$ and $b_j(t)$ where $j > o$ is equivalent to knowing all $A_j(t)$ in Eq. (A-10) for the perturbed flux.

We decouple the mode equations by setting $\Lambda_{nj} = o$ for $n \neq j$ in Eq. (A-11). This is a reasonable approximation in fast reactor systems where neutron generation time (Λ_{oo}) is of the order of 10^{-7}. Further, we consider systems with no external source ($Q_n(t)=o$) and neglect small terms $\beta_i S_{ni}(t)$ and $\delta_{ni}(t)$ in Eqs. (A-11 and A-12). With these approximations, the mode kinetics equations can be written as

$$\Lambda_{nn} \frac{\partial A_n(t)}{\partial t} + \left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right) A_n(t) - \sum_{i=1}^{6} \lambda_i C_{ni}(t) = \rho_n(t) \qquad \text{(A-23)}$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i C_{ni}(t) - \beta_i A_n(t) = o \text{ for } i = 1,6 \qquad \text{(A-24)}$$

when a small perturbation is introduced in a critical reactor such that $[b_j(t)/b_o(t)] \ll 1$ and $b_o(t) \approx 1.0$, it can be shown that Eqs. (A-23 and A-24) become the usual point kinetics equations for the amplitude function N(t) with n=o. When the inserted reactivity is small ($\rho_o < 0.5\$$), prompt jump approximation[10,11] may be used to evaluate $A_n(t)$. In the prompt jump approximation, we set $\Lambda_{nn} \frac{\partial A_n(t)}{\partial t} = o$ and the mode kinetics equations become

$$\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right) A_n(t) - \sum_{i=1}^{6} \lambda_i C_{ni}(t) = \rho_n(t) \qquad \text{(A-25)}$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i C_{ni}(t) - \beta_i A_n(t) = o \quad \text{for } i = 1,6 \qquad \text{(A-26)}$$

Formally, we may write Eq. (A-25) in the form

$$A_n(t) = \frac{\rho_n(t) + \sum_{i=1}^{6} \lambda_i C_{ni}(t)}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)}$$

$$= \frac{T_n'(t)}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)} \qquad \text{(A-27)}$$

The coupled set of equations (A-25 and A-26) can be solved analytically in the one delayed neutron family assumption.

For n > o and i = 1,

$$\frac{dA_n(t)}{dt} + \frac{\lambda}{\alpha_o} A_n(t) = \frac{\lambda\, \rho_n(t) + \frac{d\rho_n}{dt}}{\left(\frac{1}{k_n} - \frac{1}{k_o}\right) \alpha_o} \tag{A-28}$$

where

$$\alpha_o = 1 + \frac{\beta}{\left(\frac{1}{k_n} - \frac{1}{k_o}\right)}$$

and λ and β are one delayed neutron family parameters. Using the integrating factor $e^{\frac{\lambda t}{\alpha_o}}$, the solution of Eq. (A-28) may be written as

$$A_n(t) = \frac{e^{-\frac{\lambda t}{\alpha_o}}}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)} \left[B + \int_0^t dt' \; e^{\frac{\lambda t'}{\alpha_o}} \left[\lambda \rho_n(t') + \frac{d\rho_n(t')}{dt'}\right]\right.$$

$$= \frac{T_n'(t)}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)} \tag{A-29}$$

where B is the integration constant.

From observation of Eq. (A-27) and Eq. (A-29), it can be seen that $A_n(t)$ is inversely proportional to the sum of the n th mode eigenvalue separation from the fundamental and the effective delayed neutron fraction.

Since $A_n(t) = N(t)\, b_n(t)$, the expansion coefficient $b_n(t)$ is given by

$$b_n(t) = \frac{T_n'(t)}{N(t)} \frac{1}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)}$$

$$= \frac{T_n(t)}{\left(\frac{1}{k_n} - \frac{1}{k_o} + \beta\right)} \tag{A-30}$$